

Field strength correlators in QCD with dynamical fermions*

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We determine, by numerical simulations on a lattice, the gauge-invariant two-point correlation functions of the gauge field strengths in the QCD vacuum with four flavours of dynamical staggered fermions.

The gauge-invariant two-point correlators of the gauge field strengths in the QCD vacuum are defined as

$$\mathcal{D}_{\mu\rho,\nu\sigma}(x) = \langle 0 | \text{Tr} \{ G_{\mu\rho}(x) S G_{\nu\sigma}(0) S^\dagger \} | 0 \rangle, \quad (1)$$

where $G_{\mu\rho} = gT^a G_{\mu\rho}^a$ is the field-strength tensor and $S = S(x, 0)$ is the Schwinger phase operator needed to parallel-transport the tensor $G_{\nu\sigma}(0)$ to the point x .

These correlators govern the effect of the gluon condensate on the level splittings in the spectrum of heavy $Q\bar{Q}$ bound states [1–3]. They are the basic quantities in models of stochastic confinement of colour [4–6] and in the description of high-energy hadron scattering (see Ref. [7] and references therein).

A numerical determination of the correlators on the lattice already exists in the *quenched* (i.e., pure-gauge) $SU(2)$ theory [8], and also in the *quenched* $SU(3)$ theory [9,10]. Here we present results obtained in *full* QCD, i.e., by including the effects of dynamical fermions [11]. Four flavours of *staggered* fermions and the Wilson action for the pure-gauge sector have been used. The determination has been done at two different values of the quark mass.

In the Euclidean theory, the most general parametrization of the correlators is the following [4–6]:

$$\begin{aligned} \mathcal{D}_{\mu\rho,\nu\sigma}(x) = & (\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\sigma}\delta_{\rho\nu}) [\mathcal{D}(x^2) + \mathcal{D}_1(x^2)] \\ & + (x_\mu x_\nu \delta_{\rho\sigma} - x_\mu x_\sigma \delta_{\rho\nu} + x_\rho x_\sigma \delta_{\mu\nu} - \\ & x_\rho x_\nu \delta_{\mu\sigma}) \frac{\partial \mathcal{D}_1(x^2)}{\partial x^2}, \end{aligned} \quad (2)$$

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where \mathcal{D} and \mathcal{D}_1 are invariant functions of x^2 . It is also convenient to define the following quantities:

$$\begin{aligned} \mathcal{D}_{\parallel} &\equiv \mathcal{D} + \mathcal{D}_1 + x^2 \frac{\partial \mathcal{D}_1}{\partial x^2}, \\ \mathcal{D}_{\perp} &\equiv \mathcal{D} + \mathcal{D}_1. \end{aligned} \quad (3)$$

On the lattice we can define an operator $\mathcal{D}_{\mu\rho,\nu\sigma}^L$, which is proportional to $\mathcal{D}_{\mu\rho,\nu\sigma}$ in the naïve continuum limit, i.e., when the lattice spacing $a \rightarrow 0$ [9,10]:

$$\mathcal{D}_{\parallel,\perp}^L(\hat{d}a) \underset{a \rightarrow 0}{\sim} a^4 \mathcal{D}_{\parallel,\perp}(d^2 a^2) + \mathcal{O}(a^6). \quad (4)$$

However, the naïve continuum limit of Eq. (4) is spoiled by the presence of lattice artefacts, i.e., renormalization effects from lattice to continuum due to the short-range fluctuations at the scale of the UV cutoff. In order to remove these artefacts we adopt the same technique used in Refs. [9,10]. The basic idea is to remove the effects of short-range fluctuations on large distance correlators by a local *cooling* procedure [12,13].

We have measured the correlations on a $16^3 \times 24$ lattice at distances d ranging from 3 to 8 lattice spacings and at $\beta = 5.35$ ($\beta = 6/g^2$, where g is the coupling constant). We have used a standard Hybrid Monte Carlo algorithm, in particular the so-called Φ -algorithm described in Ref. [14]. The bare quark mass was chosen to be $a \cdot m_q = 0.01$. A determination was also made at $a \cdot m_q = 0.02$.

The scale of our system is fixed by the physical value of the lattice spacing a . We shall use the following parametrization

$$a(\beta) = \frac{1}{\Lambda_F} \left(\frac{8}{25} \pi^2 \beta \right)^{231/625} \exp \left(-\frac{4}{25} \pi^2 \beta \right), \quad (5)$$

where the *scaling function* $f(\beta) = \Lambda_F \cdot a(\beta)$ is given by the usual two-loop expression for gauge

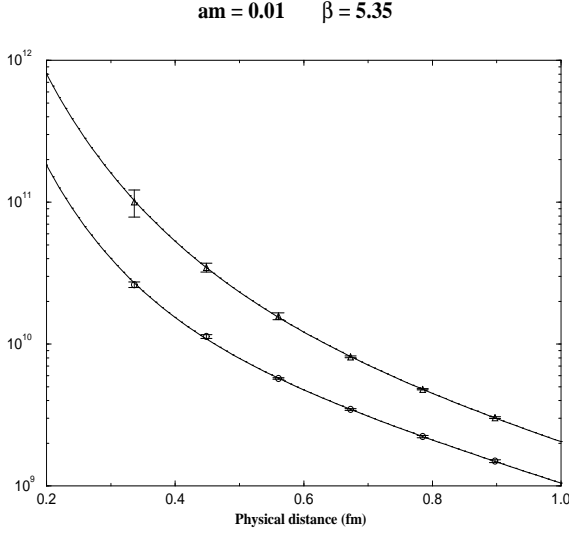


Figure 1. The functions $\mathcal{D}_\perp/\Lambda_F^4$ (upper curve) and $\mathcal{D}_\parallel/\Lambda_F^4$ (lower curve) versus physical distance, for quark mass $a \cdot m_q = 0.01$.

group $SU(3)$ and $N_f = 4$ flavours of quarks. Λ_F in Eq. (5) is an effective Λ -parameter for QCD in the lattice renormalization scheme, with $N_f = 4$ flavours of quarks. At the value of β used in our simulation the lattice spacing, extracted from the string tension or the ρ mass, is $a \simeq 0.11 \pm 0.01$ fm [15,16], so that our lattice size is approximately 2 fm and therefore safe from infrared artefacts.

In Fig. 1 we display the results for $\mathcal{D}_\parallel/\Lambda_F^4$ and $\mathcal{D}_\perp/\Lambda_F^4$ versus $d_{\text{phys}} = d \cdot a$, for a quark mass $a \cdot m_q = 0.01$. As in Ref. [10], we have tried a best fit to these data with the functions

$$\begin{aligned} \mathcal{D}(x^2) &= A_0 \exp\left(-\frac{|x|}{\lambda_A}\right) + \frac{a_0}{|x|^4} \exp\left(-\frac{|x|}{\lambda_a}\right), \\ \mathcal{D}_1(x^2) &= A_1 \exp\left(-\frac{|x|}{\lambda_A}\right) + \frac{a_1}{|x|^4} \exp\left(-\frac{|x|}{\lambda_a}\right). \end{aligned} \quad (6)$$

The continuum lines in Fig. 1 correspond to the central values of this best fit (see Ref. [11]).

The corresponding results for the quark mass $a \cdot m_q = 0.02$ are displayed in Fig. 2. From $a \cdot m_q = 0.01$ to $a \cdot m_q = 0.02$ the effective Λ_F does not change appreciably within the errors, so that we have assumed the same value of Λ_F as for $a \cdot m_q = 0.01$. Again, the continuum lines in Fig. 2 correspond to the central values of the best fit

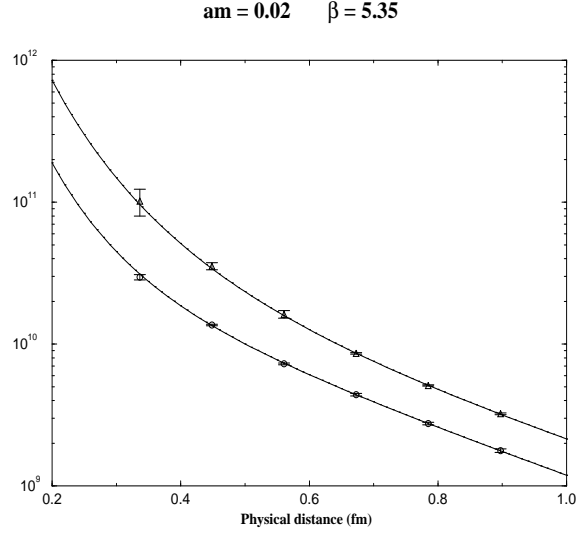


Figure 2. The functions $\mathcal{D}_\perp/\Lambda_F^4$ (upper curve) and $\mathcal{D}_\parallel/\Lambda_F^4$ (lower curve) versus physical distance, for quark mass $a \cdot m_q = 0.02$.

with the functions (6) (see Ref. [11]).

A quantity of physical interest which can be extracted from our lattice determination is the correlation length λ_A of the gluon field strengths, defined in Eq. (6): it is relevant for the description of vacuum models [4–6]. To obtain the value of λ_A in physical units, the physical value of the lattice spacing must be used. This gives, for the first quark mass $a \cdot m_q = 0.01$:

$$\lambda_A = 0.34 \pm 0.02 \pm 0.03 \text{ fm} \quad (a \cdot m_q = 0.01), \quad (7)$$

where the first error comes from our determination and the second from the uncertainty in fixing the physical scale.

Similarly, for the second quark mass $a \cdot m_q = 0.02$ we obtain:

$$\lambda_A = 0.29 \pm 0.01 \pm 0.03 \text{ fm} \quad (a \cdot m_q = 0.02). \quad (8)$$

The values (7) and (8) must be compared with the *quenched* value [9,10]

$$\lambda_A = 0.22 \pm 0.01 \pm 0.02 \text{ fm} \quad (\text{YM theory}). \quad (9)$$

The correlation length λ_A decreases by increasing the quark mass, when going from chiral to *quenched* QCD. Of course, a determination of the

physical value of λ_A should be done with more realistic values for the quark masses.

Another quantity of physical interest which can be extracted from our results is the so-called *gluon condensate*, defined as

$$G_2 \equiv \langle \frac{\alpha_s}{\pi} : G_{\mu\nu}^a G_{\mu\nu}^a : \rangle \quad (\alpha_s = \frac{g^2}{4\pi}) . \quad (10)$$

As first pointed out by Shifman, Vainshtein and Zakharov [17], it is a fundamental quantity in QCD, in the context of the sum rules.

The gluon condensate can be expressed, in terms of the parameters defined in Eq. (6), as follows [11]:

$$G_2 \simeq \frac{6}{\pi^2} (A_0 + A_1) . \quad (11)$$

At $a \cdot m_q = 0.01$ this gives, in physical units,

$$G_2 = 0.015 \pm 0.003 \stackrel{+0.006}{-0.003} \text{ GeV}^4 . \quad (12)$$

At $a \cdot m_q = 0.02$ we obtain:

$$G_2 = 0.031 \pm 0.005 \stackrel{+0.012}{-0.005} \text{ GeV}^4 . \quad (13)$$

These values should be compared with the corresponding *quenched* value [10]:

$$G_2 = 0.14 \pm 0.02 \stackrel{+0.06}{-0.05} \text{ GeV}^4 \quad (\text{YM theory}) . \quad (14)$$

As expected, the gluon condensate G_2 appears to increase with the quark mass, tending towards the (pure-gauge) value of Eq. (14). We can try to understand the dependence of G_2 on the quark masses using the following low-energy theorem [18], valid for small quark masses:

$$\frac{d}{dm_f} \langle \frac{\alpha_s}{\pi} : G_{\mu\nu}^a G_{\mu\nu}^a : \rangle = -\frac{24}{b} \langle : \bar{q}_f q_f : \rangle , \quad (15)$$

where $b = 11 - \frac{2}{3}N_f$, for a gauge group $SU(3)$ and N_f quark flavours. For $a \cdot m_q = 0.01$ we have approximately $m_f \simeq 44$ MeV [15]. Making use of the popular values for the quark condensate ($\langle \bar{q}q \rangle \simeq -0.013 \text{ GeV}^3$ [15,19]) and for the physical quark masses ($m_u \simeq 4$ MeV, $m_d \simeq 7$ MeV and $m_s \simeq 150$ MeV), we can extrapolate from the value (12) to the *physical* gluon condensate, obtaining the following estimate:

$$G_2^{(physical)} \sim 0.022 \text{ GeV}^4 . \quad (16)$$

The prediction (16) agrees with phenomenological determinations [19,20]: $G_2^{(empiric)} \simeq 0.024 \pm 0.011 \text{ GeV}^4$.

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